MATH 723 Spring 2008-09

Mathematical Neuroscience

Homework Project 3. May 10 in class.

Read:

- Course notes for Lectures 4 and 5 (as posted on the course website)
- Chapters 2 and 4 in (Izhikevich, Dynamical Neuroscience).

Problems:

1. `Saddle-node on an invariant circle bifurcation.` The following example illustrates how a saddle-node bifurcation can lead to the creation of a limit cycle. Consider a system of equations written in polar coordinates:

$$\dot{\rho} = \rho(1-\rho^2), \tag{1}$$
$$\dot{\theta} = 1+\mu-\sin\theta, \ \ \rho \geq 0, \ \theta \in S^1. \tag{2}$$

   Sketch (by hand) the phase portraits for positive and negative values of μ near 0 and for $\mu = 0$. For small $\mu > 0$, estimate how the period of the limit cycle depends on μ.

2. `The homework project for Lecture 1 'The Hodgkin-Huxley model'.` The matlab code for the numerical experiments with the HH system of the previous subsection is given in the Appendix to Lecture 1. Modify this code to study the voltage responses in the Morris-Lecar (ML) model. For the ML system, use $(V_0, n_0) = (-60.855, 0.01495)$ as the initial condition. Find the values for the amplitude and the duration of the stimulation protocols which yield sub- and supra-threshold responses, as well as trains of action potentials (AP). Repeat this numerical experiment for $\phi = 0.02$. Describe the effect of changing ϕ on the trains of APs generated during prolonged stimulations. To verify that the parameters are entered correctly, compare your numerical results with those in Figure 4 for the same values of parameters.

3. `The homework project for Lecture 2 'Approximate systems'.`

   a. Use matlab to generate phase portraits for main parameter regimes in the Morris-Lecar (ML) model. Specifically, plot the nullclines and representative trajectory converging either to the steady state or to the limit cycles for at least three different values of the applied current I: before the transition to periodic spiking, just after the transition, and farther away from the transition. For each of these regimes, plot the voltage timeseries $v(t)$.

   b. Determine whether the ML model is type I or type II. Change the value(s) of parameter(s) in this model to change its type.

   c. Repeat steps in **a.** for the modified model.

   d. Refer to the plots obtained at steps **a** and **c** to discuss two mechanisms for the action potential generation (excitability) in the Morris-Lecar model.