Name (please print): __________________________

This is a closed-book exam: no books, notebooks, notes, etc. are allowed. A cheat sheet (A4, double-sided) with formulae that might be useful on the test is permitted. Calculators are not permitted. The work must be done independently. For a full or partial credit, please justify your answers and show all of your work.

This exam contains 5 problems. Please read through the entire exam before beginning, in order to make sure that you have all the pages and in order to gauge the relative difficulty of each question. Budget your time accordingly.
1. Find a unit normal vector to the surface

\[ 2xz^2 - 3xy - 4x = 7 \]

at the point \( P = (1, -1, 2) \). Write an equation of the tangent plane to \( S \) at \( P \).
2. Evaluate the line integral $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ for

$$\mathbf{F} = [e^x, e^{-y}, e^z]$$

and

$$C : (t, t^2, t), \ t \in [0, 1].$$
3. Let \( u(x, y) = e^x \cos y + x^2 - y^2 + 3x. \)

A) Show that \( u(x, y) \) is a harmonic function.

B) Find \( v(x, y) \) such that

\[
f(z) = u(x, y) + iv(x, y), \quad z = x + iy
\]

is an analytic function.
4. Find all solutions of

\[ a. \ e^z = 2 + 2i \quad b. \ \cos z = 3i. \]
5. Evaluate $\int_C f(z)dz$ for the following data

5-a) $f(z) = \Re z^2$, $C$ is the unit circle oriented counterclockwise;

5-b) $f(z) = ze^{z^2}$, $C$ is the segment joining 1 and i.

5-c) $\int_C \frac{dz}{1-z^2}dz$, $C$ is the circle $|z+1| = 1$ oriented counterclockwise.
6.

6-a) Evaluate \( \sin \left( 4\pi - \frac{\pi i}{2} \right) \).

6-b) Find the principal value of \((1 + i)^{-1+i}\).
7. Let

\[ v(x, y, z) = 6xy \mathbf{i} + (3x^2 - 3y^2z^2) \mathbf{j} - 2y^3z \mathbf{k}. \]

Find a potential function \( \phi(x, y, z) \) such that

\[ v(x, y, z) = \nabla \phi(x, y, z), \]

or show that no such function exists.
8. Evaluate

$$\oint_C (y - \sin x) \, dx + \cos x \, dy,$$

where $C$ is the boundary of the triangle with vertices

$$(0, 0), \ (0.5\pi, 0), \ (0.5\pi, 1),$$

oriented counterclockwise.