Problem 1. Let

\[ A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 5 \\ 1 & 5 \\ 3 & 4 \end{bmatrix}. \]

Compute (a) \( A^T B \), (b) \( A^{-1} B \), (c) \( (B^T B)^{-1} \).
Problem 2. Let

\[ A = \begin{bmatrix} -8 & 5 & -2 & 0 \\ -5 & 2 & 1 & -2 \\ 9 & -11 & 7 & -3 \\ 19 & -9 & 7 & 1 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}. \]

(a) Find \( \text{Col} \ A \).

(b) Find \( \text{Null} \ A \).

(c) Determine whether \( \vec{w} \) is in \( \text{Col} \ A \), or \( \text{Null} \ A \), or both.
Problem 3. Let \( p_1(t) = 1 + t^2 \), \( p_2(t) = t - 3t^2 \), \( p_3(t) = 1 + t - 3t^2 \).

(a) Use coordinate vectors to show that polynomials \( p_1, p_3, p_3 \) form a basis for the vector space \( \mathcal{P}_2 \);

(b) Consider the basis \( \mathcal{B} = \{ p_1, p_2, p_3 \} \) for \( \mathcal{P}_2 \). Find a polynomial \( q \in \mathcal{P}_2 \), given that \( [q]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \).
Problem 4. (a) Find the eigenvalues and eigenvectors of the matrix

\[
A = \begin{bmatrix}
4 & 0 & 0 \\
0 & 4 & 0 \\
1 & 0 & 2
\end{bmatrix};
\]

(b) Diagonalize \( A \), i.e., represent it as \( A = PDP^{-1} \) where \( P \) is an invertible matrix, and \( D \) is a diagonal matrix;

(c) Verify that \( AP = PD \).
Problem 5. Let $T: \mathcal{P}_2 \to \mathcal{P}_3$ be the transformation that maps a polynomial $p(t)$ into the polynomial $(t + 5)p(t)$.

(a) Find the image of $p(t) = 2 - t + t^2$.

(b) Show that $T$ is a linear transformation.

(c) Find the matrix for $T$ relative to bases $\{1, t, t^2\}$ and $\{1, t, t^2, t^3\}$.
Problem 6. Find the complex eigenvalues and eigenvectors of

\[ A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}. \]