Let \( f(z) \) be holomorphic in an open set \( G \), and let a circle \( C \) be contained in \( G \) together with its interior. Then, by the Cauchy formula for a circle,

\[
f(z) = \frac{1}{2\pi i} \int_C \frac{f(\zeta)}{\zeta - z} \, d\zeta,
\]

where \( C \) is oriented counterclockwise and \( z \) is in the interior of \( C \). Fix \( z_0 \) in the interior of \( C \). It will be shown that \( f \) has a power series expansion with center \( z_0 \).

Let \( r_0 \) be the distance from \( z_0 \) to \( C \). Then, for each \( z \) with \(|z - z_0| < r_0\), and any \( \zeta \) on \( C \), we have a geometric series expansion

\[
\frac{1}{\zeta - z} = \frac{1}{(\zeta - z_0) - (z - z_0)} = \frac{1}{\zeta - z_0} \frac{1}{1 - \frac{z - z_0}{\zeta - z_0}} = \sum_{n=0}^{\infty} \frac{(z - z_0)^n}{(\zeta - z_0)^{n+1}}.
\]

Hence

\[
f(\zeta) = \sum_{n=0}^{\infty} \frac{f(z_0)}{(\zeta - z_0)^{n+1}} (z - z_0)^n, \quad \zeta \in C, \ |z - z_0| < r_0.
\]

For each \( z \) with \(|z - z_0| < r_0\) and each \( 0 < r < 1 \), the power series on the right converges uniformly in \( \zeta \) in the region \(|z - z_0| \leq r\).

Integrating counterclockwise over \( C \), we find that

\[
f(z) = \frac{1}{2\pi i} \int_C \frac{f(\zeta)}{\zeta - z} \, d\zeta = \sum_{n=0}^{\infty} c_n (z - z_0)^n, \quad |z - z_0| < r_0,
\]

where

\[
c_n = \frac{f^{(n)}(z_0)}{n!} = \frac{1}{2\pi i} \int_C \frac{f(\zeta)}{(\zeta - z_0)^{n+1}} \, d\zeta, \quad n = 0, 1, 2, \ldots.
\]

Thus at each point \( z \in G \), \( f \) has a power series representation and

\[
f^{(n)}(z) = \frac{n!}{2\pi i} \int_C \frac{f(\zeta)}{(\zeta - z)^{n+1}} \, d\zeta, \quad n = 0, 1, 2, \ldots.
\]