

A THREE-WAY DUEL

Three gunfighters of the Wild West got together. Mister K is a perfect shooter and never misses his target. Mister L is a good shooter, he hits with a probability of 80%. Mister H is not nearly as good and hits a modest 50% of the time. Mister K, L, and H agree to take turns shooting at each other. The order is predetermined by a random permutation of their names. Each participant is assumed to adopt the best strategy. The duel is to be continued until two of the participants are no more (the bullet supply is unlimited).

What are the overall chances of K, L, and H to survive?

Let us denote by P the probability of survival. For instance, $P(L | LH)$ is the probability that Mr L will survive a duel with Mr H, if Mr L has the first shot.

Some preliminary calculations are in order:

$$\begin{aligned} P(K | KL) &= 1 \quad \text{and} \quad P(L | KL) = 0. \\ P(L | LK) &= \frac{4}{5} \quad \text{and} \quad P(K | LK) = \frac{1}{5}. \\ P(K | KH) &= 1 \quad \text{and} \quad P(H | KH) = 0. \\ P(K | HK) &= \frac{1}{2} \quad \text{and} \quad P(H | HK) = \frac{1}{2}. \end{aligned}$$

$$P(L | LH) = \frac{4}{5} + \frac{1}{5} \cdot P(L | HL) = \frac{4}{5} + \frac{1}{5} \cdot \frac{1}{2} \cdot P(L | LH), \quad \text{so} \quad P(L | LH) = \frac{8}{9} \quad \text{and} \quad P(H | LH) = \frac{1}{9}$$

$$P(L | HL) = \frac{4}{9} \quad \text{and} \quad P(H | HL) = \frac{5}{9}$$

There are six possible initial turn arrangements to consider: HKL, HLK, KHL, KLH, LKH, LHK. We will only examine one, KLH.

Mr K takes out Mr L with his first shot, so that $P(L | KLH) = 0$.

Then Mr H shoots at Mr K, $P(K | KLH) = \frac{1}{2}$.

If Mr H happens to miss, then things are settled with Mr K's next shot, $P(H | KLH) = \frac{1}{2}$.

Can you find the total probability of survival for each participant?

The answer is $P(K) = \frac{29}{120} \approx 0.24$, $P(L) = \frac{14}{45} \approx 0.31$, $P(H) = \frac{161}{360} \approx 0.45$.