Let a random variable \( X \) be uniformly distributed in the interval \( 0 < x < \theta \).

Consider two simple hypotheses, based on a single observation of \( X \),

\[ H_0 : \quad \theta = 1 \quad \text{and} \quad H_1 : \quad \theta = 1.1. \]

The likelihood ratio function is

\[ \ell(x) = \frac{P(X = x | H_0)}{P(X = x | H_1)} = \frac{f_0(x)}{f_1(x)} = \begin{cases} 1.1, & 0 < x < 1, \\ 0, & 1 \leq x < 1.1. \end{cases} \]

Note that \( \ell(x) \) is a decreasing step function defined on \( (0, 1.1) \) with range \( \{0, 1.1\} \).

Under \( H_0 \), it is impossible to observe an \( x \) in \( [1, 1.1) \); under \( H_1 \), this chance is \( 1/11 \).

Given a critical value \( c \), the likelihood ratio test is

\[ \ell(x) < c : \quad \text{reject} \ H_0 \]
\[ \ell(x) = c : \quad \text{reject} \ H_0 \quad \text{with probability} \ q \]
\[ \ell(x) > c : \quad \text{accept} \ H_0 \]

The borderline probability of rejection \( q \) may depend on \( x \) (and on \( c \)).

For any \( c \) with \( 0 < c < 1.1 \), the significance level of the test is

\[ \alpha = P(\ell(X) < c \mid \theta = 1) = 0, \]

and the power of the test is \( 1 - \beta = P(H_1 \mid \theta = 1) = P(\ell(X) < c \mid \theta = 1.1) = 1/11 \).

Thus the odds of rejection when \( H_0 \) is true are zero, but the odds of acceptance when \( H_0 \) is false are high.

The cases where \( c \) belongs to the range of \( \ell(x) \) are more delicate.

If \( c = 0 \), we have \( \alpha = P(\ell(X) = 0 \mid \theta = 1) = 0 \) and \( 1 - \beta = P(H_1 \mid \theta = 1.1) = 10/11 \int_0^{1.1} q(x)dx \).

If \( c = 1.1 \), we have \( \alpha = P(H_1 \mid \theta = 1) = \int_0^1 q(x)dx \) and \( 1 - \beta = P(H_1 \mid \theta = 1.1) = 10/11 \int_0^1 q(x)dx + 1/11 \).

For instance, if \( c = 1.1 \) and \( q(x) = 0.5 \), then \( \alpha = 0.5 \) and \( 1 - \beta = 6/11 \).