Let $a_0, a_1, a_2, \ldots$ be a sequence satisfying the following conditions:

\[ a_0 = 1, a_1 = 0, \]
\[ a_n = 4a_{n-1} - 3a_{n-2} + 1, \quad n = 2, 3, \ldots \]

The task is to find a formula for $a_n$.

To begin, let us find the general solution of the corresponding homogeneous problem

\[ (H) \quad a_n = 4a_{n-1} - 3a_{n-2}. \]

Try a solution of the form $a_n = x^n$:

\[ x^n = 4x^{n-1} - 3x^{n-2} \]
\[ x^2 = 4x - 3 \quad \text{after division by } x^{n-2} \]
\[ x^2 - 4x + 3 = 0 \]
\[ (x - 1)(x - 3) = 0 \]
\[ x = 1, \ x = 3 \quad \text{characteristic roots} \]

So $a_n = 1^n = 1$ and $a_n = 3^n$ both satisfy relation (H).

Since the relation is linear, $a_n = A \cdot 1 + B \cdot 3^n$ satisfies (H) for any constants $A$ and $B$.

Next consider the nonhomogeneous problem

\[ (N) \quad a_n = 4a_{n-1} - 3a_{n-2} + 1. \]

We only need to recognize one solution of (N). How to find it?

Note that the differences $\Delta_n = a_n - a_{n-1}$ satisfy the relation $\Delta_n = 3\Delta_{n-1} + 1$,

which is a first-order nonhomogeneous recurrence. Clearly, if $\Delta_n$ is constant,

it must have a value of $-1/2$. Hence $a_n = -n/2$ is a solution of (N).

Any other solution of (N) may be written in the form

\[ a_n = [A + B \cdot 3^n] + [-n/2], \]

where $A$ and $B$ are constants. It remains to choose $A$ and $B$ to meet the initial conditions:

\[ a_0 = A + B \cdot 3^0 - 0/2 = 1, \]
\[ a_1 = A + B \cdot 3^1 - 1/2 = 0. \]

A quick calculation gives $A = 5/4$ and $B = -1/4$.

Therefore, as is easy to check, $a_n = 5/4 - 3^n/4 - n/2, \quad n \in \mathbb{N}$, is the solution sought.