DREXEL ANALYSIS SEMINAR

October 7, 2022

2-3 PM, Korman 245

Speaker: Hein van der Holst (Georgia State)

Title: The Strong Maximum Nullity of Directed Graphs

Abstract: For a directed graph $D = (V, A)$ with vertex-set $V = \{v_1, v_2, \ldots, v_n\}$ in which we permit parallel arcs and loops, we denote by $Q(D)$ the set of all $n \times n$ real matrices $A = [a_{i,j}]$ such that if $a_{i,j} \neq 0$ and $i \neq j$, then there is at least one arc from vertex v_i to vertex v_j, if $a_{i,j} = 0$ and $i \neq j$, then either there is no arc from vertex v_i to vertex v_j or there are at least two arcs from vertex v_i to vertex v_j, and if $a_{i,i} = 0$, then there is a loop at vertex v_i. A matrix $A \in Q(D)$ has the Asymmetric Strong Arnold Property (ASAP) if there is no nonzero $n \times n$ real matrix $X = [x_{i,j}]$ such that $x_{i,i} = 0$ for $i = 1, \ldots n$, $x_{i,j} = 0$ whenever there is an arc from vertex v_i to vertex v_j, and $X^\top A = 0$ and $AX^\top = 0$. There always exists a matrix $A \in Q(D)$ satisfying the ASAP. For a directed graph D, denote by $\zeta(D)$ the maximum nullity of any $A \in Q(D)$ satisfying the ASAP.
In this talk, we give characterizations of the class of directed graphs D with $\zeta(D) = 0$ and of the class of directed graphs D with $\zeta(D) \leq 1$. We do this in terms of forbidden substructures (minors). We also discuss the signed version.

This joint work with Marina Arav, Louis A. Deaett, H. Tracy Hall, and Derek Young.