Drexel Analysis Seminar

September 29, 2023

12-1 PM, Korman 245

Speaker: Prateek Vishwakarma (University of Regina, Canada)

Title: Inequalities for totally nonnegative matrices: Gantmacher–Krein, Karlin, and Laplace

Abstract: A real linear combination of products of minors which is nonnegative over all totally nonnegative (TN) matrices is called a determinantal inequality for these matrices. It is referred to as multiplicative when it compares two collections of products of minors and additive otherwise. Set theoretic operations preserving the class of TN matrices naturally translate into operations preserving determinantal inequalities in this class. We introduce index-row (and index-column) operations that act directly on all determinantal inequalities for TN matrices, and yield further inequalities for these matrices. These operations assist in revealing novel additive inequalities for TN matrices embedded in the classical identities due to Laplace [*Mem. Acad. Sciences Paris* 1772] and Karlin (1968). In particular, for any square TN matrix A, these derived inequalities generalize – to every i^{th} row of A and j^{th} column of adj$A$ – the classical Gantmacher–Krein fluctuating inequalities (1941) for $i = j = 1$. Furthermore, our index-row/column operations reveal additional undiscovered fluctuating inequalities for TN matrices.

The introduced index-row/column operations naturally birth an algorithm that can detect certain determinantal expressions that do not form an inequality for TN matrices. However, the algorithm completely characterizes the multiplicative inequalities comparing products of pairs of minors. Moreover, the underlying index-row/column operations add that these inequalities are offshoots of certain "complementary/higher" ones. These novel results seem very natural, and in addition thoroughly describe and enrich the classification of these multiplicative inequalities due to Fallat–Gekhtman–Johnson [*Adv. Appl. Math.* 2003] and later Skandera [*J. Algebraic Comb.* 2004] (and revisited by Rhoades–Skandera [*Ann. Comb.* 2005, *J. Algebra* 2006]).

This is joint work with Shaun M. Fallat.